Novel vortex-transform for high frequency modulated patterns

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Abstract: A novel vortex-transform is proposed. This transform allows for generating complex-valued functions from modulated intensity patterns, including high frequency components from modulation, without the generation of unstable phase singularities. From these complex-valued functions it is possible to obtain intensity and pseudo-phase maps to analyze the intensity recordings without the necessity of phase retrieval techniques. The intensity and pseudo-phase maps obtained by using this transform preserve the modulation structure onto the intensity and phase modulo 2π maps, including stable phase singularities.

OCIS codes: (070.0070) Fourier optics and signal processing; (100.2960) Image analysis; (260.6042) Singular optics.

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1. Introduction

In many fields of optics the modulated patterns have being used along the time for numerous purposes; this is because of the information that can be easily and selectively retrieved from intensity recordings [1]. Applications based on the modulation of images have being used for many years and refined over time due to its utility and reliability. Some contributions take into advantage the Fourier analysis of the amplitude-modulation, as in the cases of the pattern comparison by interferometric methods or the positioning and alignment of setups, like the use of Moiré patterns in nanolithography alignment [2, 3]. Some other employs modulation to retrieve phase modulo 2π information for a wide range of purposes. With both amplitude-modulation and phase retrieval capability it is possible to obtain additional information, as in the case of applications to analyze fringe images, synthesize multiple phases in digital holography, and in digital speckle interferometry [4–6], among many others.

In the recent years a particular interest in the singular optics field has been rising. This field of optics also uses the modulation to retrieve information that is not available otherwise; in particular, in some proposals, the optical singularities from N-pinhole interferometer are characterized and employed to measure the angular momentum from astrophysical light [7, 8]. This N-pinhole system is also used in the generation of high-order optical vortices when pinholes are arranged in a spiral line around the origin [9]. However, these applications rely on ad hoc phase modulo 2π from fields, and this information is not always available or easy to obtain.

There exist several transforms to obtain a complex signal from intensity distributions; the obtained signals allows for assigning a pseudo-phase map to a given distribution, and processing it like a field where ad hoc phase modulo 2π is available. In particular, in the optical vortex metrology (OVM) field, some applications rely in the generation of complex signals from speckle or speckle-like distributions by using the Hilbert, Riesz or Laguerre-Gauss transforms [10], among others. By its definition, the analytical signal obtained by using a Hilbert Transform is anisotropic in 2D patterns, given that it uses a Heaviside step function as filter, which in turns implies that the spatial resolution of measurements depends on evaluation direction. The Riesz and Laguerre-Gauss transforms are the isotropic counterparts of the Hilbert transform; these alternatives use a spiral pure phase plate function as filter, and for this reason any section crossing the filter's origin can be associated with a signum function with a π phase gap. The advantage of Laguerre-Gauss transform over Riesz transform relies on the inclusion of a band-pass filter which eliminates any DC component of the pattern and attenuate high frequency components that lead to unstable phase singularities [11–14]. But any of these transforms are not suitable to synthesize modulated patterns.

In modulated patterns, the inclusion of high frequency components given by fringe systems superposition lead to the generation of plenty unstable phase singularities when using Riesz transform to obtain the complex signal from intensity distribution, and, on the other hand, the properties of the modulation are diminished by Laguerre-Gauss transform band-pass filter. For this reason we propose a novel transform to preserve the high frequency components from modulated distributions without the generation of unstable phase singularities in the pseudo-phase retrieval from complex-valued signal representation.

2. Vortex transform for modulated patterns

2.1 Filter description

To analyze modulated patterns with high frequency information, a novel vortex transform is proposed. This transform allows for obtaining a complex valued function from an intensity modulated pattern, taking in advantage the increase of phase singularities without the inclusion of unstable singularities, which is achieved by using a linear operator transform with a filter including the high frequency components from these patterns. In fact, from a modulated intensity distribution $g(x, y)$ it is possible to relate an isotropic complex signal $\tilde{g}(x, y)$ by using a linear transform given by

$$\tilde{g}(\mathrm{x,y}) = \int_{-\infty}^{\infty}\int H\left(f_x, f_y\right)\cdot G\left(f_x, f_y\right)\exp\left[2\pi i\left(f_x x + f_y y\right)\right]df_x df_y \qquad (1)$$

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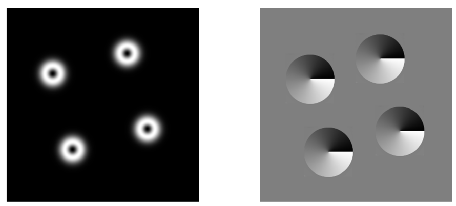
where, f_x and f_y are frequency coordinates, the function $G(f_x, f_y)$ is defined as the Fourier spectrum of the original intensity distribution $g(x, y)$, and $H(f_x, f_y)$ is the Fourier spectrum of the linear superposition of Laguerre-Gauss filters in a function $h(\mathrm{x}, \mathrm{y})$ defined as:

$$\mathrm{h}(\mathrm{x},\mathrm{y}) = \sum_{j=i}^{n} \left(i\pi^2\omega^4\right)\left[r_j \exp\left(-\pi^2 r_j^2 \omega^2\right)\exp\left(i\alpha_j\right)\right] \quad (2)$$

where $r_j = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2}$, $\alpha_j = \arctan\left(\left(y - y_j\right)/\left(x - x_j\right)\right)$ and ω is a parameter to set the width of the band-pass filter. For each main diffraction halo in $G(f_x, f_y)$ there is a corresponding Laguerre-Gauss filter contributing to the summation over n elements in Eq. (2), whose individual location is determined by the respective spot in the halo. In the following we will call this transform the Vortex Transform for Modulated Patterns (VT-MP).

To serve as an example from filter in spatial coordinates, Fig. 1 presents the corresponding intensity and phase from the $h(x, y)$ function when synthesizing modulated patterns from the interference of the light diffracted by a four pupil apertures mask. It is observed that filter is compound by four filters with doughnut-like amplitudes and whose phases are spiral pure phase plate functions. Each of this filters are located where the pupil apertures employed to generate the modulated pattern are placed.



(a) (b)

Fig. 1. Doughnut-like amplitudes and spiral pure phase plate functions for the proposed filter when using Eq. (2) with a four pupil aperture mask: In (a) the amplitude and in (b) the phase modulo 2π.

2.2 Experimental setup

In order to depict the vortex transform for modulated patterns (VT-MP) outputs, Young's fringe systems from the diffraction of multi-aperture masks, and modulated speckle patterns intensity distributions where obtained by using the setups from Figs. 2(a) and 2(b), respectively. For Young's fringes case, a plane wave of wavelength λ is diffracted by a n-apertures pupil mask with aperture's diameter d, symmetrically distributed over a circumference of radius R. The intensity pattern is recorded in a plane located at a distance z from the mask, which is large enough to satisfy the far-field condition of the Fraunhoffer diffraction. A diffuser is attached to the mask in the latter setup in order to obtain modulated speckle patterns.

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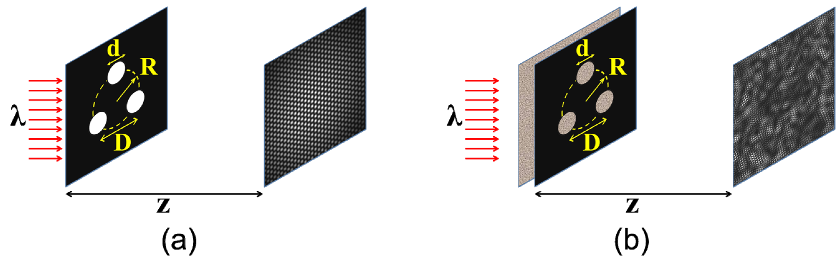

Fig. 2. Setups to generate modulated intensity patterns. (a) A plane wave impinges over a multi-aperture mask, and is diffracted through a distance z, where the observation plane is located. . (b) A diffuser is attached to the multi-aperture mask in the latter setup to generate modulated speckle patterns.

3. Experimental results and discussion

Table 1 presents the simulated results for multi-aperture masks containing 4, 6, 8 and 12 apertures, the diameter of each aperture is 13 pixels, located over a circumference of radius 400 pixels, the pixel width and height is 10 μm. These simulations are for distributions 4096x4096 pixels^2 in size, propagated a distance of 2.75 m using a wavelength of 632.8 nm. The first row depicts the intensity from the interference pattern, and the second row the respective phase modulo 2π. The next two rows present the results obtained by applying the VT-MP to the intensity maps in row 1. The intensity and pseudo-phase map obtained are shown in row 3 and 4, respectively. It should be noted that both the intensity and pseudo-phase maps preserve the modulation structure, but include the phase singularities from the usage of the proposed transform.

In order to establish the suitability from the VT-MP, a comparison between the proposed transform and the Laguerre-Gauss transform where conducted. Table 2 presents the results of a modulated intensity pattern obtained by the light diffracted by a mask with 12 apertures, where the diameter of each aperture is 13 pixels, located over a circumference of radius 185 pixels, the pixel width and height is 10 μm. These simulations are for distributions 2048x2048 pixels^2 in size, propagated a distance of 1.45 m using a wavelength of 632.8 nm. In the left column the intensity and pseudo-phase maps obtained when using VT-MP are depicted, Laguerre-Gauss transform maps are presented in the right column. In these results the information missed when using Laguerre-Gauss transform become apparent, VT-MP on the other hand preserve this information, taking in advantage the high-frequency components from modulation. The superposition of the pseudo-intensity results obtained by both transforms is depicted in Fig. 3. It can be easily observed that VT-MP intensity map includes the Laguerre-Gauss transform information (green spots) and the additional information that the latter cannot resolve.

The experimental results of the intensity and pseudo-phase maps obtained by using the VT-MP from a modulated speckle patterns are presented in Fig. 4. A diffuser is superposed to a 4 apertures mask as depicted in Fig. 2(b); the diameter of each aperture is 2.25 mm, located over a circumference of radius 1 cm; a CMOS camera with a pixel size of 3.5 μm was employed, the sensor area was 1280x960 pixels^2, later cropped to square image of 960x960 pixels^2 for calculation simplicity. The propagation distance is 0.49 m and a wavelength of 532 nm is used. This speckle patterns present an inner modulation structures where phase singularities appear when using vortex related transforms. Note that the proposed transform VT-MP allows locating singularities inside the modulation structures, even if the modulation is a local phenomenon. It should be noted that VT-MP is suitable not only to analyze simulated results but also experimental recordings, when using different parameter sets as presented in this section.

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Table 1. Simulated results for VT-MP applied to a plane wave diffracted by a multi-aperture mask

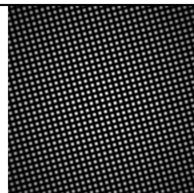
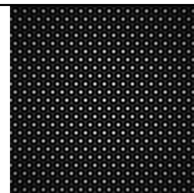
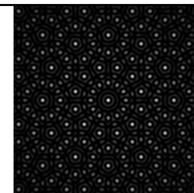
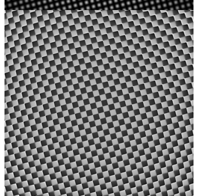
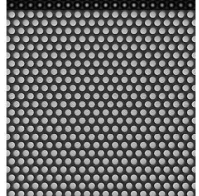
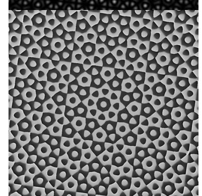
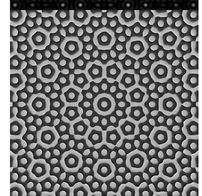
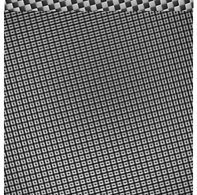
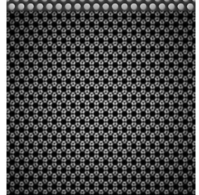
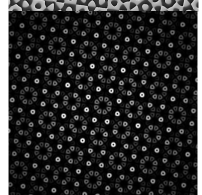
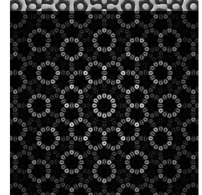
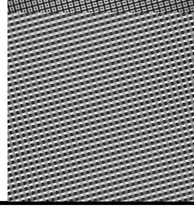
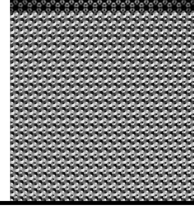
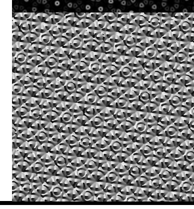
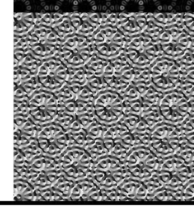
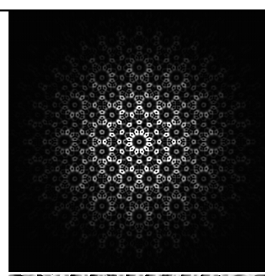
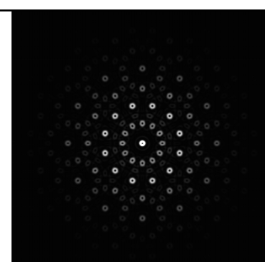
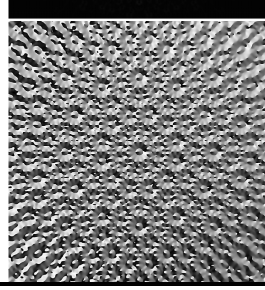
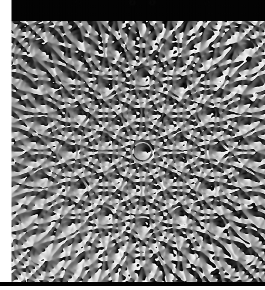
	4 Apertures	*6 Apertures*	*8 Apertures*	*12 Apertures*
Intensity				
Phase modulo 2π				
VT-MP Intensity				
VT-MP Phase				

Table 2. Simulated results for VT-MP and Laguerre-Gauss transform applied to the light diffracted by 12 multi-aperture mask

	VT-MP	*Laguerre-Gauss*
Pseudo-Intensity		
Pseudo-phase		

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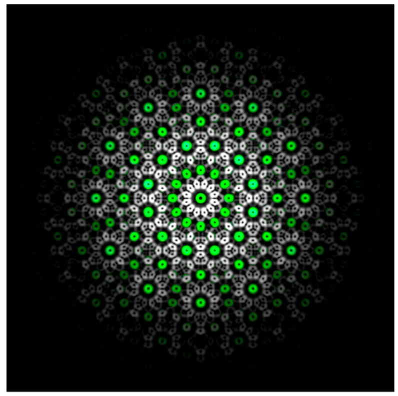

Fig. 3. Superposition of the intensity map obtained by using VT-MP and Laguerre-Gauss. The coincident information is depicted in green

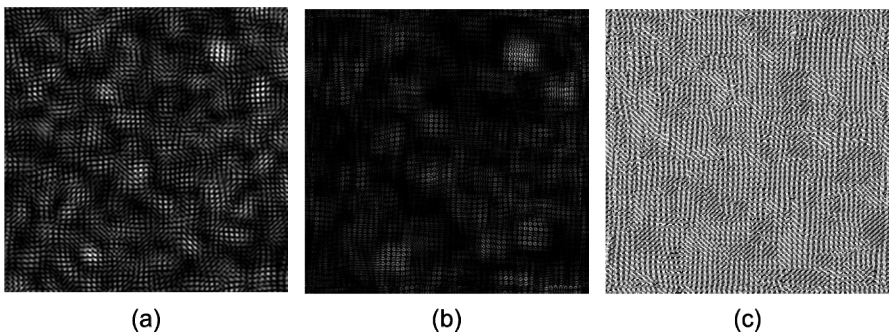

Fig. 4. Experimental results for modulated speckle patterns, by using the VT-MP. In (a) the recorded speckle intensity, in (b) the VT-MP intensity distribution, and in (c) the respective pseudo-phase map.

4. Conclusions

In this paper we proposed a novel vortex-transform for modulated patterns. By using this transform, it is possible to generate complex-valued functions from modulated intensity distribution. The comparison between Laguerre-Gauss transform and the proposed transform allows demonstrating the suitability from VP-MT to resolve high-frequency components of modulated intensity patterns, which are not resolved by Laguerre-Gauss transform. The intensity and pseudo-phase maps obtained by using VP-MT when analyzing results concerning to a plane wave diffracted by multi-apertures mask with and without a diffuser are presented. We analyzed both experimental images and simulated results. Also, we foresee VT-MP as a useful tool for analyzing intensity records with high frequency components from modulation in the singular optics field.

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